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**ABSTRACT**

In this study, we investigate the inventory model over a period of fixed planning for a deteriorating item having a selling price demand rate in which shortages are allowed and are partially backlogged. The results are verified with an numerical example.

**KEYWORDS:** Deteriorating items, Inventory model, Partial backlogging, Selling price demand, Shortage.

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**INTRODUCTION**

Many people have studied inventory models for items which is perishable such as electronic components, food stuffs, drugs and fashion goods. In real situations goods may be of time dependent, price dependent and stock dependent. The important role in inventory system is selling price. In fact the inventoried item is continuously depleting due to the demand effects and deterioration of stock level. In the last few years, great attention was given to inventory lot-sizing models using deterioration.

Inventory problems with time variable demand patterns has been investigated by several researchers . Donaldson [2] solved the classical inventory problem for a linear trend in demand over a finite time horizon with no shortage. However, Donaldson's solution procedure was computationally complicated. Misra et al. [7] presented a easy procedure for the economic order quantity model for the cases of increasing or decreasing demand with linear trend. In the above models, the possibilities of shortages and deterioration inventory were left out of consideration. Mishra and Singh [8, 9] constructed an inventory model for ramp-type demand, deteriorating items with time dependent salvage value and shortages and deteriorating inventory model for time dependent demand and holding cost and with partial back logging. Hung [4] investigated an inventory model with generalized type demand, deterioration and back order rates. Milu Acharya & Smrutirekha[6] developed an inventory model for deteriorating items with time dependent demand under partial backlogging.

In this paper we developed an inventory model with constant deterioration and selling price demand. Shortages are allowed and are partially backlogged.

The assumption and notation used in this paper are as follows:

- i. The demand rate is price dependent, i. e.  $D(t)=a-p$
- ii. The replenishment rate is infinite, thus replenishment is instantaneous.
- iii.  $I(t)$  is the level of inventory at time  $t$ ,  $0 \leq t \leq T$ .
- iv.  $T$  is the length of the cycle.
- v.  $\theta$  is the constant deteriorating rate,  $0 < \theta < 1$ .
- vi.  $t_1$  is the time when the inventory level reaches zero.
- vii.  $Q$  is the ordering quantity per cycle.
- viii.  $A$  is the fixed ordering cost per order.

- ix.  $C_1$  is the cost of each deteriorated item.
- x.  $C_2$  is the inventory holding cost per unit per unit of time.
- xi.  $C_3$  is the shortage cost per unit per unit of time.
- xii.  $S$  is the maximum inventory level for the ordering cycle, such that  $S=I(0)$ .
- xiii.  $TC$  is the total average cost per unit time with the condition  $t \leq T$ .

### MATHEMATICAL FORMULATION

Here we consider the inventory model with deteriorating price dependent demand rate. When the inventory level attains its maximum replenishment occurs at time  $t=0$ . The inventory level reduces due to demand and deterioration from  $t=0$  to  $t_1$ . When the inventory level reaches zero at  $t_1$ , then at the time interval  $(t_1, T)$  shortage is allowed to occur and is completely backlogged. The total number of backlogged items is replaced by the different replenishment. Using the notations and assumptions, the behavior of inventory system at any time is given by the following differential equations:

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t) \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -D(t) \quad t_1 \leq t \leq T \quad (2)$$

With boundary condition  $I(0)=S, I(t_1)=0$

The solutions of equations (1) & (2) are

$$I(t) = \frac{a-p}{\theta} [e^{\theta(t_1-t)} - 1] \quad 0 \leq t \leq t_1 \quad (3)$$

$$I(t) = (a-p) [t_1 - T] \quad t_1 \leq t \leq T \quad (4)$$

The beginning inventory level can be computed as

$$S=I(0) = \frac{a-p}{\theta} [e^{\theta t_1} - 1] \quad (5)$$

The total number of items which perish in the interval  $[0, t_1]$ , say  $D$  is

$$\begin{aligned} D &= S - \int_0^{t_1} I(t) dt \\ &= \frac{a-p}{\theta} [e^{\theta t_1} - 1] - (a-p) t_1 \end{aligned} \quad (6)$$

The total number of inventory carried during the interval  $[0, t_1]$ , say  $H$  is

$$\begin{aligned} H &= \int_0^{t_1} I(t) dt \\ &= \frac{a-p}{\theta^2} [e^{\theta t_1} - 1] - \frac{a-p}{\theta} t_1 \end{aligned} \quad (7)$$

The total shortage quantity during the interval  $[t_1, T]$ , say  $B$  is

$$\begin{aligned} B &= - \int_{t_1}^T I(t) dt \\ &= (a-p) (T^2 + t_1^2 - 2t_1T) \end{aligned} \quad (8)$$

Then the average total cost per unit time when  $t_1 \leq T$  can be given by

$$\begin{aligned} TC &= \frac{1}{T} [A + C_1 D + C_2 H + C_3 B] \\ &= \frac{1}{T} [A + C_1 \left( \frac{a-p}{\theta} [e^{\theta t_1} - 1] - (a-p) t_1 \right) + C_2 \left( \frac{a-p}{\theta^2} [e^{\theta t_1} - 1] - \frac{a-p}{\theta} t_1 \right) + C_3 ((a-p) (T^2 + t_1^2 - 2t_1T))] \end{aligned} \quad (9)$$

The necessary condition that the total cost is to be minimized is  $\frac{dTC}{dt_1} = 0$  is

$$(a-p) \left[ \left( C_1 + \frac{C_2}{\theta} \right) (e^{\theta t_1} - 1) + 2C_3(t_1 - T) \right] = 0$$

### NUMERICAL EXAMPLE

Consider an inventory system with parameters in proper unit  $A=1000, a=25, p=400, C_1=0.5, C_2=0.02, C_3=0.3, T=2$  we get  $t_1=1.91926$  and  $TC=490.86798$

### SENSITIVITY ANALYSIS

$\theta$	$t_1$	TC
0.01	1.91926	490.86798
0.02	1.90302	490.92873
0.03	1.93364	490.93255
0.04	1.87052	490.80490
0.05	1.85428	490.69688
0.06	1.83806	490.55811
0.07	1.82188	490.38824
0.08	1.80573	490.18874

## CONCLUSION

In this model we have developed an inventory model with time dependent selling price and partial backlogging for deteriorating item. This model shows that when the deterioration cost increases the total cost decreases.

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